Acceleration Forces Aboard NASA KC-135 Aircraft During Microgravity Maneuvers

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Introduction

DURING the typical 30-s microgravity flight maneuvers of the NASA KC-135 aircraft, it is essential that pitch and power requirements are strictly adhered to in order to provide a laboratory space within an aircraft that is free of inertial accelerations. Thrust must precisely offset drag, which at first diminishes, then increases with passage through the apex of the parabola. During the maneuver, the pitch angle of the aircraft changes from approximately +45 deg to a −45 deg, causing a torque at every station except at the aircraft's center of mass.

Mathematical Derivation

The effects of the pitch change on the quality of the microgravity profile have been mathematically determined for any station in the aircraft at any time during the maneuver. It can be shown that the pitch change accelerations are small and that the KC-135 is an excellent microgravity simulator.

To obtain zero-gravity conditions, an aircraft's center of mass must have a vertical component of acceleration equal to the acceleration of gravity. Therefore,

$$a_x^c = 0 \tag{1}$$

$$a_y^c = -g \tag{2}$$

From these conditions, differential equations are solved to obtain the velocity and position equations.

$$v_x^c = V_{x0} \tag{3}$$

$$v_y^c = V_{y0} - gt \tag{4}$$

$$x^c = X_0^c - V_{x0}t \tag{5}$$

$$y^c = Y_0^c + V_{y0}t - \tfrac{1}{2}gt^2 \tag{6}$$

where V_{0x}, V_{0y}, x_0^c, and y_0^c are the initial conditions of velocity and position, respectively, and v_x, v_y, x^c, and y^c are the instantaneous velocities and positions at the center of mass.

Although the net acceleration at the aircraft's center of mass can be reduced to zero, positions away from the center of mass will experience an acceleration due to the rotation of the aircraft from the initial "nose-up" pitch to the final "nose-down" pitch.

For a displacement ψ from the aircraft's center of mass at an attitude described by the angle α, the equations are

$$x' = x^c + \psi\cos\alpha = X_{0c} + V_{0x}t + \psi\cos\alpha \tag{7}$$

$$y' = y^c + \psi\sin\alpha = Y_{0c} + V_{0y}t - \tfrac{1}{2}gt^2 + \psi\sin\alpha \tag{8}$$

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Differentiating with respect to time we obtain

$$V_x' = V_{0x} - \psi\dot\alpha\sin\alpha \tag{9}$$

$$v_y' = V_{0y} - gt + \psi\dot\alpha\cos\alpha \tag{10}$$

Differentiating again with respect to time we obtain

$$a_x' = -\psi(\ddot\alpha\sin\alpha + \dot\alpha^2\cos\alpha) \tag{11}$$

$$a_y' = -g + \psi(\ddot\alpha\cos\alpha - \dot\alpha^2\sin\alpha) \tag{12}$$

The net acceleration with respect to the center of mass is

$$a_\psi = a_x' i + a_y' j - a_y^c j = a_x i + a_y' j + gj$$

$$= \psi[(-\ddot\alpha\sin\alpha - \dot\alpha^2\cos\alpha)i + (\ddot\alpha\cos\alpha - \dot\alpha^2\sin\alpha)j] \tag{13}$$

The magnitude of the net acceleration is

$$|a_\psi| = \psi[(-\ddot\alpha\sin\alpha - \dot\alpha^2\cos\alpha)^2 + (\ddot\alpha\cos\alpha - \dot\alpha^2\sin\alpha)^2]^{1/2}$$

$$= \psi[\ddot\alpha^2(\sin^2\alpha + \cos^2\alpha) + \dot\alpha^4(\cos^2\alpha + \sin^2\alpha)$$

$$+ 2\ddot\alpha\dot\alpha^2(\sin\alpha\cos\alpha - \cos\alpha\sin\alpha)]^{1/2}$$

$$= \psi[\ddot\alpha^2 + \dot\alpha^4]^{1/2} \tag{14}$$

The α may be written in terms of aircraft velocity

$$\tan\alpha = \frac{V_y}{V_x} = \frac{V_{y0} - gt}{V_{x0}} \tag{15}$$

Differentiating gives us the following

$$\frac{d}{dt}\left(\frac{V_{y0} - gt}{V_{x0}}\right) = \frac{d}{dt}(\tan\alpha) = \dot\alpha\sec^2\alpha \tag{16}$$

Rewriting and putting $\dot\alpha$ on the left-hand side gives

$$\dot\alpha = \left[\frac{d}{dt}\left(\frac{V_{y0} - gt}{V_{x0}}\right)\right]\Big/(\sec^2\alpha)$$

$$= \left[\frac{d}{dt}\left(\frac{V_{y0} - gt}{V_{x0}}\right)\right]\Big/(\tan^2\alpha + 1)$$

$$= \frac{(-g/V_{x0})}{[(V_{y0} - gt)/V_{x0}]^2 + 1} = \frac{-V_{x0}g}{V_{x0}^2 + (V_{y0} - gt)^2} \tag{17}$$

A second differentiation of α with respect to time is

$$\ddot\alpha = \frac{d\dot\alpha}{dt} = \frac{d}{dt}\left(\frac{-V_{x0}g}{V_{x0}^2 + (V_{y0} - gt)^2}\right)$$

$$= \frac{[V_{x0}^2 + (V_{y0} - gt)]\dfrac{d(V_{x0}g)}{dt} - (-V_{x0}g)\left(\dfrac{d}{dt}[V_{x0}^2 - (V_{y0} - gt)^2]\right)}{[V_{x0}^2 + (V_{y0} - gt)^2]}$$

$$= \frac{-2V_{x0}g^2(V_{y0} - gt)}{[V_{x0}^2 + (V_{y0} - gt)^2]^2} \tag{18}$$

The magnitude of the net acceleration may then be described in terms of the aircraft's velocity as follows:

$$|a_\psi| = \psi(\ddot\alpha^2 + \dot\alpha^4)^{1/2} \tag{19}$$

$$|a_\psi| = \psi\left\{\left[\frac{-2V_{x0}g^2(V_{y0}-gt)}{[V_{x0}^2+(V_{y0}-gt)^2]^2}\right]^2 + \left[\frac{-V_{x0}g}{V_{x0}^2+(V_{y0}-gt)^2}\right]^4\right\}^{1/2}$$

$$= \psi\left\{\frac{4V_{x0}^2g^4(V_{y0}-gt)^2}{[V_{x0}^2+(V_{y0}-gt)^2]^4} + \frac{V_{x0}^4g^4}{[V_{x0}^2+(V_{y0}-gt)^2]^4}\right\}^{1/2}$$

$$= \frac{\psi V_{x0}g^2}{[V_{x0}^2+(V_{x0}-gt)^2]^2}\,[V_{x0}^2+4(V_{y0}-gt)^2]^{1/2} \tag{20}$$

The maximum net acceleration caused by the aircraft's rotation may be determined by calculating $(\mathrm{d}|a_\psi|)/\mathrm{d}t$ and then setting it to zero.

$$\frac{\mathrm{d}|a_\psi|}{\mathrm{d}t} = \frac{[V_{x0}^2+(V_{y0}-gt)^2]^2\,\frac{\mathrm{d}}{\mathrm{d}t}\,\{\psi V_{x0}g^2[V_{x0}^2+4(V_{y0}-gt)^2]^{1/2}\}}{[V_{x0}^2+(V_{y0}-gt)^2]^4}$$

$$- \frac{\psi V_{x0}g^2[V_{x0}^2+4(V_{y0}-gt)^2]^{1/2}\,\frac{\mathrm{d}}{\mathrm{d}t}\,\{[V_{x0}^2+(V_{y0}-gt)^2]^2\}}{[V_{x0}^2+(V_{y0}-gt)^2]^4}$$

$$= \frac{[V_{x0}^2+(V_{y0}-gt)^2][\psi V_{x0}g^2]\{(1/2)/[V_{x0}^2+4(V_{y0}-gt)^2]^{1/2}\}[8(V_{y0}-gt)][-g]}{[V_{x0}^2+(V_{y0}-gt)^2]^4}$$

$$- \frac{2[V_{x0}^2+(V_{y0}-gt)^2]\{\psi V_{x0}g^2[V_{x0}^2+4(V_{y0}-gt)^2]^{1/2}\}[2(V_{y0}-gt)(-g)]}{[V_{x0}^2+(V_{y0}-gt)^2]^4} \tag{21}$$

By setting the differential equal to zero, the equation can be factored as follows:

$$0 = [V_{x0}^2+(V_{y0}-gt)^2][V_{x0}^2+4(V_{y0}-gt)^2]^{1/2}$$

$$-[V_{x0}^2+4(V_{y0}-gt)^2]^{1/2}$$

$$= V_{x0}^2+(V_{y0}-gt)^2-[V_{x0}^2+4(V_{y0}-gt)^2]$$

$$= -3(V_{y0}-gt)^2$$

$$= V_{y0}-gt \quad \text{or} \quad t = V_{y0}/g \tag{22}$$

If $V_{y0} = 483$ ft/s, then the maximum acceleration occurs at $t = 15$ s. The maximum net acceleration for $\psi = 30$ ft (a position 30 ft from the aircraft's center of mass) and the initial conditions of $V_{x0} = V_{y0} = 483$ ft/s ($V_{y0} = gt$) is

$$|a_{30\ \mathrm{ft}}|_{\substack{(V_{y0}-gt)\\ \text{at } t=15\ \mathrm{s}}} = \frac{\psi g^2}{V_{x0}^2} = \frac{30(32.17)^2}{(483)^2} = 0.1331\ \mathrm{ft/s^2}$$

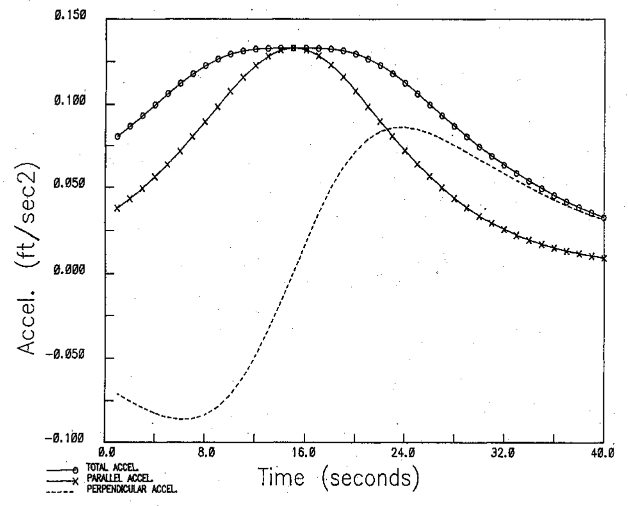

Fig. 1 Graph of data for an object 30 ft forward of the center of mass, with $V_{x0} = V_{y0} = 483$ ft/s.

The net acceleration may be divided into a component along or parallel to the flight path ($a_{\parallel\psi}$) and a component perpendicular to the flight path but in the plane of the flight path ($a_{\perp\psi}$).

$$|a_{\perp\psi}| = \psi\,\frac{\mathrm{d}^2(\alpha)}{\mathrm{d}t} = \psi\ddot{\alpha} = \frac{-2\psi V_{x0}g^2(V_{y0}-gt)}{[V_{x0}^2+(V_{y0}-gt)^2]^2} \tag{23}$$

$$|a_{\parallel\psi}| = \psi\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2 = \psi\dot{\alpha}^2 = \frac{\psi V_{x0}^2g^2}{[V_{x0}^2+(V_{y0}-gt)^2]^2} \tag{24}$$

Table 1 contains a tabulated set of numbers for the total, parallel, and perpendicular components of acceleration at a point 30 ft from the center of mass in the KC-135. These results are graphed in Fig. 1.

Table 1 Calculations of net acceleration for a displacement forward from the aircraft's center of mass of 30 ft

Time	Total net acceleration, ft/s².	Component perpendicular to flight path	Component parallel to flight path
0.00	0.0744	−0.0665	0.0332
1.00	0.0805	−0.0709	0.0380
2.00	0.0868	−0.0752	0.0434
3.00	0.0933	−0.0791	0.0494
4.00	0.0998	−0.0825	0.0562
5.00	0.1063	−0.0850	0.0637
6.00	0.1123	−0.0863	0.0719
7.00	0.1179	−0.0860	0.0806
8.00	0.1227	−0.0837	0.0896
9.00	0.1266	−0.0791	0.0988
10.00	0.1295	−0.0719	0.1077
11.00	0.1314	−0.0619	0.1159
12.00	0.1325	−0.0493	0.1229
13.00	0.1329	−0.0344	0.1284
14.00	0.1330	−0.0177	0.1319
15.00	0.1331	−0.0002	0.1331
16.00	0.1330	0.0173	0.1319
17.00	0.1329	0.0340	0.1285
18.00	0.1325	0.0490	0.1231
19.00	0.1315	0.0616	0.1161
20.00	0.1296	0.0717	0.1079
21.00	0.1267	0.0790	0.0990
22.00	0.1228	0.8370	0.0899
23.00	0.1180	0.0860	0.0808
24.00	0.1125	0.0863	0.0721
25.00	0.1064	0.0851	0.0639
26.00	0.1000	0.0826	0.0564
27.00	0.0935	0.0792	0.0496
28.00	0.0870	0.0753	0.0435
29.00	0.0806	0.0710	0.0381
30.00	0.0745	0.0666	0.0333
31.00	0.0687	0.0622	0.0292
32.00	0.0633	0.0579	0.0256
33.00	0.0582	0.0537	0.0224
34.00	0.0534	0.0498	0.0197
35.00	0.0492	0.0461	0.0173
36.00	0.0452	0.0426	0.0152
37.00	0.0416	0.0394	0.0134
38.00	0.0383	0.0364	0.0119
39.00	0.0353	0.0337	0.0105
40.00	0.0325	0.0311	0.0093